

Grade 5 Math Unit 3- Number and Operations - Fractions

UNIT OVERVIEW

In Grade 5 instruction should center around 3 Critical Focus Areas. This unit addresses **Critical Focus Area #1, Developing fluency with addition and subtraction of fractions and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions).**

(See Connections for explanation)

This unit will address the following clusters:

- Use equivalent fractions as a strategy to add and subtract fractions
- Apply and extend previous understandings of multiplication and division to multiply and divide fractions

STANDARDS

CC_Common Core State Standards - Mathematics (2010) - Grade 5

Domain 5.NF Number and Operations-Fractions

Cluster Statement: *Use equivalent fractions as a strategy to add and subtract fractions.*

Standard 5.NF.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.

Standard 5.NF.2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.

Cluster Statement: *Apply and extend previous understandings of multiplication and division to multiply and divide fractions.*

Standard 5.NF.3 Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

Standard 5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

5.NF.4.a Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$.

5.NF.4.b Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

Standard 5.NF.5 Interpret multiplication as scaling (resizing), by:

5.NF.5.a Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.

5.NF.5.b Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.

Standard 5.NF.6 Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

Standard 5.NF.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.

5.NF.7.a Interpret division of a unit fraction by a non-zero whole number, and compute such quotients.

5.NF.7.b Interpret division of a whole number by a unit fraction, and compute such quotients.

5.NF.7.c Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and

CONTENT ELABORATIONS

5.NF.1	<p>5.NF.1 builds on the work in fourth grade where students add fractions with like denominators. In fifth grade, the example provided in the standard has students find a common denominator by finding the product of both denominators. For $\frac{1}{3} + \frac{1}{6}$, a common denominator is 18, which is the product of 3 and 6. This process should be introduced using visual fraction models (area models, number lines, etc.) to build understanding before moving into the standard algorithm.</p> <p>Student should apply their understanding of equivalent fractions developed in fourth grade and their ability to rewrite fractions in an equivalent form to find common denominators. They should know that multiplying the denominators will always give a common denominator but may not result in the smallest denominator. MP.2, MP.4, MP.7 should be emphasized.</p>
5.NF.2	<p>5.NF.2 refers to number sense, which means students understanding of fractions as numbers that lie between whole numbers on a number line. Number sense in fractions also includes moving between decimals and fractions to find equivalents, also being able to use reasoning such as $\frac{7}{8}$ is greater than $\frac{3}{4}$ because $\frac{7}{8}$ is missing only $\frac{1}{8}$ and $\frac{3}{4}$ is missing $\frac{1}{4}$ so $\frac{7}{8}$ is closer to a whole. Also, students should use benchmark fractions to estimate and examine the reasonableness of their answers. Example: $\frac{5}{8}$ is greater than $\frac{6}{10}$ because $\frac{5}{8}$ is $\frac{1}{8}$ larger than $\frac{1}{2}$ ($\frac{4}{8}$) and $\frac{6}{10}$ is only $\frac{1}{10}$ larger than $\frac{1}{2}$ ($\frac{5}{10}$). MP.1 - MP.8 should be emphasized.</p>
5.NF.3	<p>5.NF.3 extends work of partitioning a number line from grades 3-4. Students need ample experiences to explore the concept that a fraction is a way to represent the division of two quantities. Students are expected to demonstrate their understanding using concrete materials, drawing models and explaining their thinking when working with fractions in multiple contexts. They read $\frac{3}{5}$ as "three fifths" and after many experiences with sharing problems, learn that $\frac{3}{5}$ can also be interpreted as "3 divided by 5".</p> <p><i>Examples:</i></p> <ul style="list-style-type: none">- Ten team members are sharing 3 boxes of cookies. How much of a box will each student get? When working this problem a student should recognize that the 3 boxes are being divided into 10 groups, so s/he is seeing the solution to the following equation, $10 \times n = 3$ (10 groups of some amount is 3 boxes) which can also be written as $n = \frac{3}{10}$. Flashcard of a math symbol for Division 10. Using models or diagram, they divide each box into 10 groups, resulting in each team member getting $\frac{3}{10}$ of a box.- Two after school clubs are having pizza parties. For the Math Club, the teacher will order 3 pizzas for every 5 students. For the student council, the teacher will order 5 pizzas for every 8 students. Since you are in both groups, you need to decide which party to attend. How much pizza would you get at each party? If you want to have the most pizza, which party should you attend? <p>MP. 1, MP.2, MP.3, MP.4, MP.5, MP.7 should be emphasized.</p>
5.NF.4	<p>Students need to develop a fundamental understanding that the multiplication of a fraction by a whole number could be represented as repeated addition of a unit fraction. For example: $2 \times \frac{1}{4} = \frac{1}{4} + \frac{1}{4}$</p> <p>This standard extends students' work of multiplication from earlier grades. In fourth grade, students worked with recognizing that a fraction such as $\frac{3}{5}$ actually could be represented as 3 pieces that are each one-fifth ($3 \times \frac{1}{5}$)</p> <p>In fifth grade, students are expected to multiply fractions including proper fractions, improper fractions, and mixed numbers. They multiply fractions efficiently and accurately as well as solve problems in both contextual and non-contextual situations.</p> <p>This standard references both the multiplication of a fraction by a whole number and the multiplication of two fractions. Visual fraction models (area models, tape diagrams, number lines) should be used and created by students during their work with this standard.</p> <p>As they multiply fractions such as $\frac{3}{5} \times 6$, they can think of the operation in more than one way.</p> <ul style="list-style-type: none">- $3 \times (\frac{6}{5})$ Flashcard of a math symbol for Division 5) or $(3 \times \frac{6}{5})$- (3×6) Flashcard of a math symbol for Division 5 or 18 Flashcard of a math symbol for Division 5 ($\frac{18}{5}$) <p>Students create a story problem for $\frac{3}{5} \times 6$ such as,</p> <ul style="list-style-type: none">- Isabel had 6 feet of wrapping paper. She used $\frac{3}{5}$ of the paper to wrap some presents. How much does she have left?- Every day Tim ran $\frac{3}{5}$ of mile. How far did he run after 6 days? (Interpreting this as $6 \times \frac{3}{5}$) <p>MP.1-MP.8 should be emphasized.</p>

5.NF.5a	<p>5.NF.5a calls for students to examine the magnitude of products in terms of the relationship between two types of problems. This extends the work of 5.OA.1. <i>Example:</i> Mrs. Jones teaches in a room that is 60 ft. wide and 40 ft. long. Mr. Thomas teaches in a room that is half as wide, but has the same length. How do the dimensions and area of Mr. Thomas' classroom compare to Mrs. Jones' room? Draw a picture to prove your answer.</p>
5.NF.5b	<p>5.NF.5b asks students to examine how numbers change when we multiply by fractions. Students should have ample opportunities to examine both cases in the standard: a) when multiplying by a fraction greater than 1, the number increases and b) when multiplying by a fraction less than one, the number decreases. This standard should be explored and discussed while students are working with 5.NF.4 and should not be taught in isolation. <i>Example:</i> Mrs. Bennett is planting two flower beds. The first flower bed is 5 meters long and $\frac{6}{5}$ meters wide. The second flower bed is 5 meters long and $\frac{5}{6}$ meters wide. How do the areas of these two flower beds compare? Is the value of the area larger or smaller than 5 square meters? Draw pictures to prove your answer. MP.2, MP.4, MP.6, MP.7 should be emphasized.</p>
5.NF.6	<p>5.NF.6 builds on all of the work done in this cluster. Students should be given ample opportunities to use various strategies to solve word problems involving the multiplication of a fraction by a mixed number. This standard could include fraction by a fraction, fraction by a mixed number or mixed number by a mixed number. <i>Example:</i> There are $2\frac{1}{2}$ bus loads of students standing in the parking lot. The students are getting ready to go on a field trip. $\frac{2}{5}$ of the students on each bus are girls. How many buses would it take to carry only the girls? <i>Student 1:</i> I drew 3 grids and 1 grid represents 1 bus. I cut the 3rd grid in half and I marked out the right half of the 3rd grid, leaving $2\frac{1}{2}$ grids. I then cut each grid into fifths, and shaded two-fifths of each grid to represent the number of girls. When I added up the shaded pieces, $\frac{2}{5}$ of the 1st and 2nd bus were both shaded, and $\frac{1}{5}$ of the last bus was shaded = $\frac{5}{5} = 1$ whole bus. <i>Student 2:</i> $2\frac{1}{2} \times \frac{2}{5} =$ I split the $2\frac{1}{2}$ into 2 and $\frac{1}{2}$. $2 \times \frac{2}{5} = \frac{4}{5}$ $\frac{1}{2} \times \frac{2}{5} = \frac{2}{10}$ I then added $\frac{4}{5}$ and $\frac{2}{10}$. That equals 1 whole bus load. MP.1 - MP.8 should be emphasized.</p>
5.NF.7	<p>5.NF.7 is the first time that students are dividing with fractions. In fourth grade, students divided whole numbers, and multiplied a whole number by a fraction. In fifth grade, students experience division problems with whole number divisors and unit fraction dividends (fractions with a numerator of 1) or with unit fraction divisors and whole number dividends. For example, the fraction $\frac{3}{5}$ is 3 copies of the unit fraction $\frac{1}{5}$. $\frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{3}{5} = \frac{1}{5} \times 3$ or $3 \times \frac{1}{5}$. Students extend their understanding of the meaning of fractions, how many unit fractions are in a whole, and their understanding of multiplication and division as involving equal groups or shares and the number of objects in each group/share. In sixth grade, they will use this foundational understanding to divide into and by more complex fractions and develop abstract methods of dividing by fractions.</p>
5.NF.7a	<p>5.NF.7a asks students to work with story contexts where a unit fraction is divided by a non-zero whole number. Student should use various fraction models and reasoning about fractions. <i>Example:</i> You have $\frac{1}{8}$ of a bag of pens and you need to share them among 3 people. How much of the bag does each person get?</p>
5.NF.7b	<p>5.NF.7b calls for students to create story contexts and visual fraction models for division situations where a whole number is being divided by a unit fraction. <i>Example:</i> Create a story for 5 divided by $\frac{1}{6}$. Find your answer and then draw a picture to prove your answer and use multiplication to reason about whether your answer makes sense. How many $\frac{1}{6}$ are there in 5?</p>
5.NF.7c	<p>5.NF.7c students should continue to use visual fraction models and reasoning to solve these real-world problems. MP.1 - MP.8 should be emphasized.</p>

UNIT VOCABULARY

<p>fraction numerator denominator</p>	<p>simplest form equivalent fractions multiple</p>	<p>least common denominator (LCD) like fractions unlike fractions</p>
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common factors
greatest common factor (GCF)

common multiples
least common multiple (LCM)

scaling
unit fraction

BIG IDEAS

ENDURING UNDERSTANDINGS

ESSENTIALS QUESTIONS

Choose a few questions based on the needs of your students

- A fraction is another representation for division.
- Fractions are relations—the size or amount of the whole matters.
- Fractions may represent division with a quotient less than one.
- Equivalent fractions represent the same value.
- With unit fractions, the greater the denominator, the smaller the piece is.
- Pieces don't have to be congruent to be equivalent.
- Fractions and decimals are different representations for the same amounts and can be used interchangeably.

- How are factors and multiples helpful in solving problems?
- How can equivalent fractions help me add and subtract fractions?
- What strategies can be used to multiply and divide fractions?

CONNECTIONS

In Critical Focus Area #1, students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them. Students also use the meaning of fractions, of multiplication and division and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the act of dividing unit fractions by whole numbers and whole numbers by unit fractions.)

This unit is connected to **3.NF.3a-c**, **5.OA.1** and is a foundation for learning in **6.NF.1**

Standards for Mathematical Practice (SMP)

MP.1 Make sense of problems and persevere in solving them

MP.2 Reason abstractly and quantitatively

MP.3 Construct viable arguments and critique the reasoning of others

MP.4 Model with mathematics

MP.5 Use appropriate tools strategically

MP.6 Attend to precision

MP.7 Look for and make use of structure (Deductive reasoning)

MP.8 Look for and express regularity in repeated reasoning (Inductive Reasoning)

USE EQUIVALENT FRACTIONS AS A STRATEGY TO ADD AND SUBTRACT FRACTIONS

CONTENT

SKILLS

5.NF.1	Add and subtract fractions with unlike denominators.	Add and subtract fractions with unlike denominators. 1. Generate equivalent fractions to find the like denominator. 2. Use different strategies, such as number lines, area models, fractions bar or strips, to add and subtract fractions. 3. Solve addition and subtraction problems involving fractions (including mixed numbers) with like and unlike denominators using an equivalent fraction strategy.
5.NF.2	Solve word problems involving addition and subtraction of fractions referring to the same whole.	Solve word problems involving addition and subtraction of fractions referring to the same whole. 1. Generate equivalent fractions to find like denominators. 2. Solve word problems involving addition and subtraction of fractions with unlike denominators referring to the same whole (e.g. by using visual fraction models or equations to represent the problem) 3. Evaluate the reasonableness of an answer, using fractional number sense, by comparing it to a benchmark fraction.

APPLY AND EXTEND PREVIOUS UNDERSTANDINGS OF MULTIPLICATION AND DIVISION TO MULTIPLY AND DIVIDE FRACTIONS

	CONTENT	SKILLS
5.NF.3	Interpret a fraction as division of the numerator by the denominator.	Interpret a fraction as division of the numerator by the denominator. 1. Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). 2. Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers. (e.g. using visual fraction models or equations to represent the problem.) 3. Interpret the remainder as a fractional part of the problem.
5.NF.4	Multiply a fraction or whole number by a fraction.	Multiply a fraction or whole number by a fraction. 1. Multiply fractions by whole numbers. 2. Multiply fractions by fractions; including proper fractions, improper fractions and mixed numbers. 3. Interpret the product of a fraction times a whole number as total number of parts of the whole or repeated addition of a unit fraction. (for example $\frac{3}{4} \times 3 = \frac{3}{4} + \frac{3}{4} + \frac{3}{4} = \frac{9}{4}$) 4. Determine the sequence of operations that result in the total number of parts of the whole. (for example $\frac{3}{4} \times 3 = (3 \times 3)/4 = 9/4$) 5. Interpret the product of a fraction times a fraction as the total number of parts of the whole. 6. Represent problems involving multiplication of fractions using various fraction models. (e.g. area models, tape diagrams and number lines)
5.NF.4b	Find the area of a rectangle with fractional side lengths.	Find the area of a rectangle with fractional side lengths. 1. Find area of a rectangle with fractional side lengths using different strategies. (e.g., tiling with unit squares of the appropriate unit fraction side lengths, multiplying side lengths) 2. Represent fraction products as rectangular areas. 3. Justify multiplying fractional side lengths to find the area is the same as tiling a rectangle with unit squares of the appropriate unit fraction side lengths. 4. Model the area of rectangles with fractional side lengths with unit squares to show the area of rectangles.

5.NF.5a	Interpret multiplication as scaling (resizing), by comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.	Interpret multiplication as scaling (resizing), by comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. 1. Know that scaling (resizing) involves multiplication. 2. Compare the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. (For example, a 2x3 rectangle would have an area twice the length of 3.)
5.NF.5b	Explain why multiplying a given number by a fraction greater than 1 results in a product greater than the given number.	Explain why multiplying a given number by a fraction greater than 1 results in a product greater than the given number. 1. Know that multiplying whole numbers and fractions result in products greater than or less than 1 depending upon the factors. 2. Draw a conclusion multiplying a fraction greater than one will result in a product greater than the given number. 3. Draw a conclusion that when you multiply a fraction by one (which can be written as various fractions, ex $\frac{2}{2}$, $\frac{3}{3}$, etc.) the resulting fraction is equivalent. 4. Draw a conclusion that when you multiply a fraction by a fraction, the product will be smaller than the given number.
5.NF.6	Solve real world problems involving multiplication of fractions and mixed numbers.	Solve real world problems involving multiplication of fractions and mixed numbers. 1. Represent word problems involving multiplication of fractions and mixed numbers (e.g., by using visual fraction models or equations to represent the problem.) 2. Solve real world problems involving multiplication of fractions and mixed numbers.
5.NF.7	Divide unit fractions by whole numbers and whole numbers by unit fractions.	Divide unit fractions by whole numbers and whole numbers by unit fractions. 1. Know the relationship between multiplication and division 2. Interpret division of a unit fraction by a whole number and justify your answer using the relationship between multiplication and division, and by creating story problems, using visual models, and relationship to multiplication, etc. 3. Interpret division of a whole number by a unit fraction and justify your answer using the relationship between multiplication and division, and by representing the quotient with a visual fraction model. 4. Solve real world problems involving division of unit fractions by whole numbers other than 0 and division of whole numbers by unit fractions using strategies such as visual fractions models and equations.

UNIT RESOURCES

Common Core Model Curriculum

McGraw-Hill, **My Math** Chapters 8-10

Georgia Math frameworks, Grade 5, Unit 4

Manipulatives: Fraction bars, Fraction circles, Number Lines

Thinking Blocks - Model fractions in word problems : http://www.thinkingblocks.com/tb_fractions/fractions.html

NCTM Illuminations Equivalent Fractions : <http://illuminations.nctm.org/ActivityDetail.aspx?ID=80>

NLVM - Fraction Bars http://nlvm.usu.edu/en/nav/frames_asid_203_g_2_t_1.html

NLVM - Fractions - Adding http://nlvm.usu.edu/en/nav/frames_asid_106_g_2_t_1.html

NLVM - Number Line Bars http://nlvm.usu.edu/en/nav/frames_asid_180_g_2_t_1.html

NLVM Adding Fractions with unlike denominators: http://nlvm.usu.edu/en/nav/frames_asid_106_g_2_t_1.html?from=grade_g_2.html